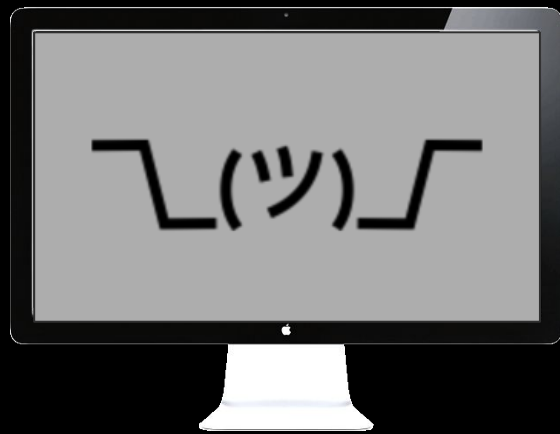
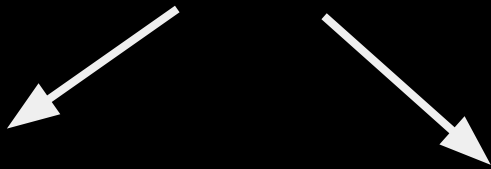
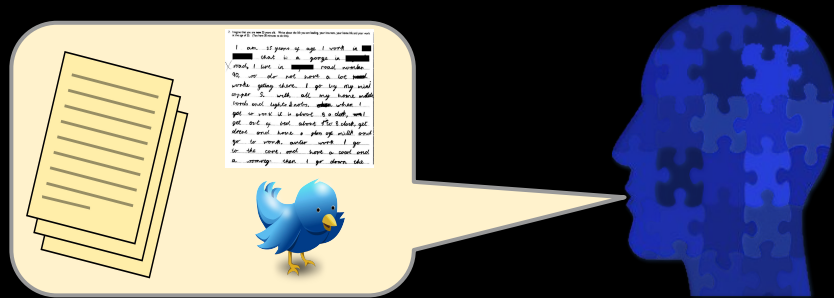


# Introduction to Natural Language Processing

CSE392 - Spring 2019  
Special Topic in CS

1. General goal for NLP and appreciation for complexity.
2. Course Topics
3. Preliminary methods



What is it like to be a computer?

The horse raced past the barn.

What is it like to be a computer?

The horse raced past the barn.

The horse raced past the barn fell.

What is it like to be a computer?

The horse raced past the barn.



The horse raced past the barn fell.



# What is it like to be a computer?

The horse raced past the barn.



The horse raced past the barn fell.



The horse **runs** past the barn.



The horse **runs** past the barn fell.



# What is it like to be a computer?

The horse raced past the barn. ✓

The horse **raced** past the barn fell. ✓

that was

The horse **runs** past the barn. ✓

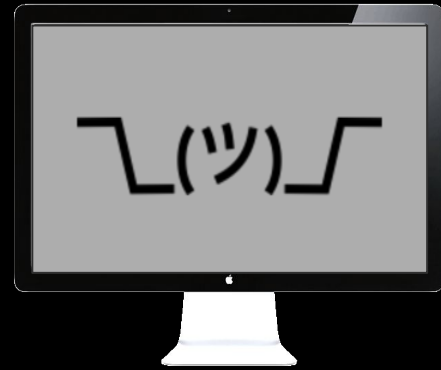
The horse **runs** past the barn fell. ✗



More empathy for the computer...

She ate the cake with the frosting.

She ate the cake with the fork.



## More empathy for the computer...

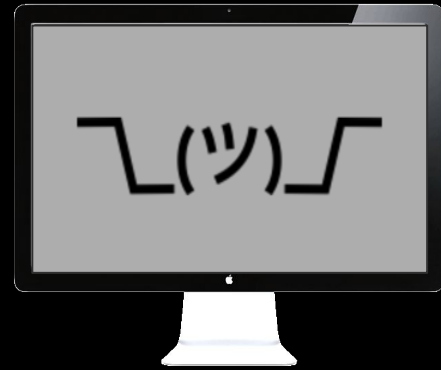
She ate the cake with the frosting.

She ate the cake with the fork.

He put the **port** on the ship.

He walked along the **port** of the steamer.

He walked along the **port** next to the steamer.



More empathy for the computer...



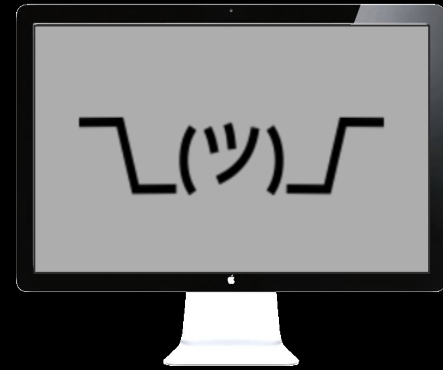
Colorless purple ideas sleep furiously. (Chomsky, 1956; “purple”=> “green”)

Fruit flies like a banana.      Time flies like an arrow.

Daddy what did you bring that book that I don't want to be  
read to out of up for?

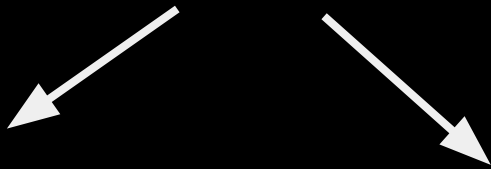
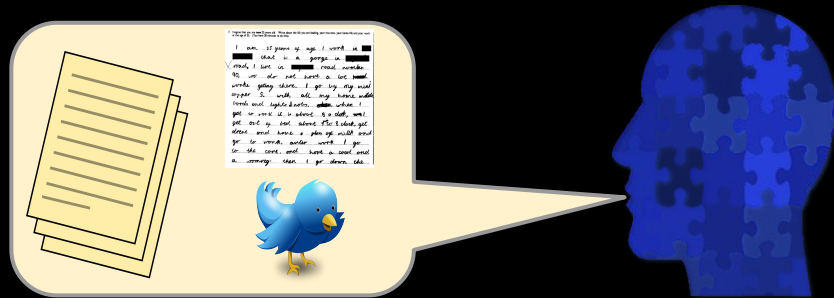
(Pinker, 1994)

More empathy for the computer...

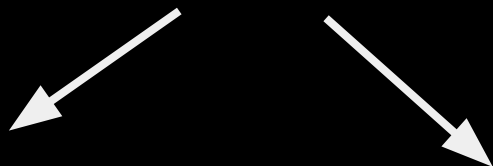
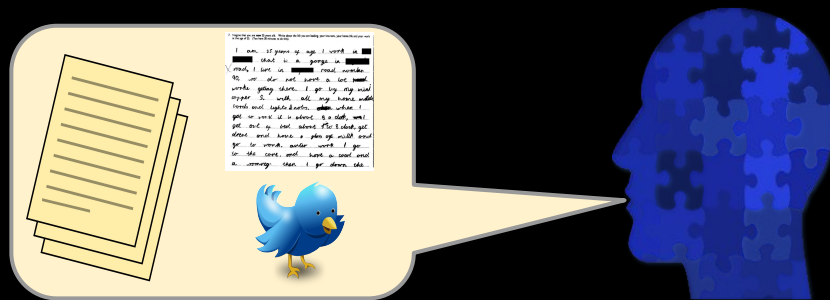


Daddy what did you bring that book that I don't want to be read to out of up for?

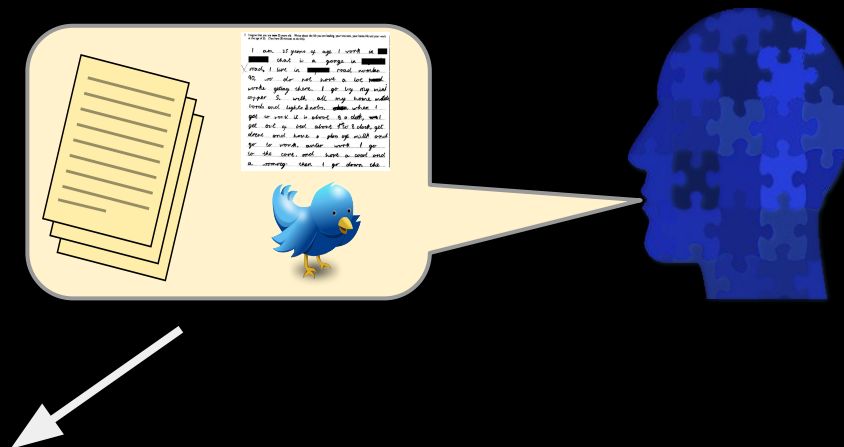
(Pinker, 1994)



# NLP's grand goal: completely understand natural language.

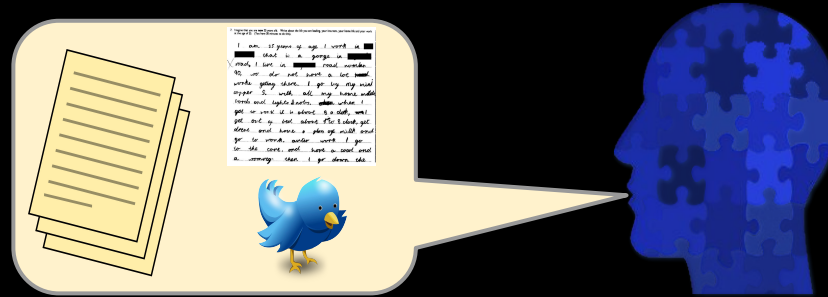


# NLP's practical applications



- Machine translation
- Automatic speech recognition
  - Personalized assistants
  - Auto customer service
- Information Retrieval
  - Web Search
  - Question Answering
- Sentiment Analysis
- *Human Language Analysis*

# NLP's practical applications



- Machine translation
- Automatic speech recognition
  - Personalized assistants
  - Auto customer service
- Information Retrieval
  - Web Search
  - Question Answering
- Sentiment Analysis
- *Human Language Analysis*

how?

(common examples)

- Machine learning:
  - Logistic regression
  - Deep learning
  - Recurrent Neural Networks
- Algorithms, e.g.:
  - Dynamic programming
  - Graph analytics
- Data science
  - Hypothesis testing



# Speech and Language Processing

An Introduction to Natural Language Processing,  
Computational Linguistics, and Speech Recognition

Third Edition draft

Daniel Jurafsky  
*Stanford University*

James H. Martin  
*University of Colorado at Boulder*

*Copyright ©2018*

Draft of September 23, 2018. Comments and typos welcome!

Week	Topics	Reading Assignment	Assignments, Exams
<b>I. Syntax</b>			
1/28	Introduction to NLP; Regular Expressions	SLP 2.1, 2.4	
2/4	Logistic Regression; POS Tagging	SLP 8.1 - 8.3	A1 Released
2/11	Language Modeling	SLP 3.1 - 3.4	A1 Due
2/18	TensorFlow; Recurrent Neural Networks	<u>TS Paper</u> , SLP 9.1 - 9.4	
2/25	Syntactic Parsing; <b>Exam 1</b>	SLP 11.1, 11.3	<b>Exam 1 (th, 2/28)</b>
<b>II. Semantics</b>			
3/4	Named Entity Recognition; Word Sense Disambiguation	SLP 6.1; C.1-C.5	
3/11	Dependency Parsing	SLP 13.1 - 13.4	A2 Released
3/18	<b>Spring Recess: No Classes</b>		A2 Due
3/25	Semantic Role Labeling; Verbal Predicates	SLP 18.1-18.3; 18.6	Team Signup
4/1	Vector Models; Neural Language Models	SLP 6.2-6.4; 7.1,7.5	
4/8	Neural LMs contd.; <b>Exam 2</b>		<b>Exam 2 (th, 4/11)</b>
<b>III. Applications</b>			
4/15	Sentiment Analysis; Human Centered NLP	<u>HovySpruit, Lynn etAl</u>	A3 Released
4/22	Differential Language Analysis	SLP 19.5,19.7,19.8 <u>Kern etAl</u>	A3 Due
4/29	TBD: Machine Translation, Speech Recog, Transformers, QA	SLP 23.1, 23.3, TBD	
5/6	TEam Project Presentations		Team Project Due
5/14 -5/22	<b>Final Exam during scheduled exam period</b>		<b>Exam 3</b>

Week	Topics	Reading Assignment	Assignments, Exams
------	--------	--------------------	--------------------

## I. Syntax

1/28	Introduction to NLP; Regular Expressions	SLP 2.1, 2.4	
2/4	Logistic Regression; POS Tagging	SLP 8.1 - 8.3	A1 Released
2/11	Language Modeling	SLP 3.1 - 3.4	A1 Due
2/18	TensorFlow; Recurrent Neural Networks	<u>TS Paper</u> , SLP 9.1 - 9.4	
2/25	Syntactic Parsing; Exam 1	SLP 11.1, 11.3	<b>Exam 1 (th, 2/28)</b>

Week	Topics	Reading Assignment	Assignments, Exams
<b>II. Semantics</b>			
3/4	Named Entity Recognition; Word Sense Disambiguation	SLP 6.1; C.1-C.5	
3/11	Dependency Parsing	SLP 13.1 - 13.4	A2 Released
3/18	Spring Recess: No Classes		A2 Due
3/25	Semantic Role Labeling; Verbal Predicates	SLP 18.1-18.3; 18.6	Team Signup
4/1	Vector Models; Neural Language Models	SLP 6.2-6.4; 7.1,7.5	
4/8	Neural LMs contd.; Exam 2		<b>Exam 2 (th, 4/11)</b>

### III. Applications

4/15	Sentiment Analysis; Human Centered NLP	<u>HovySpruit,</u> <u>Lynn_etAl</u>	A3 Released
4/22	Differential Language Analysis	SLP 19.5,19.7,19.8 <u>Kern_etAl</u>	A3 Due
4/29	TBD: Machine Translation, Speech Recog, Transformers, QA, Dialog Systems	SLP 23.1, 23.3, TBD	
5/6	Team Project Presentations		Team Project Due
5/14 -5/22	Final Exam during scheduled exam period		<b>Exam 3</b>

# Ingredients for success

The following covers the major components of the course and the estimated amount of time one might put into each if they are aiming to fully learn the material.

- **Readings:** 1 - 2 hours; 10 - 20 pages per week (best before each class)
- **Study:** 1 - 2 hours per week to review notes and looking up extra content (plus 3 to 4 hours to review before each exam)
- **Homeworks (3):** 3 to 8 hours each
- **NLP in the World (2):** 2 to 4 hours preparing each presentation
- **Team Project:** 5 to 15 for each of the last 4 weeks (ramping up)

# Course Website

<http://www3.cs.stonybrook.edu/~has/CSE392/>

# Preliminary Methods

Regular Expressions - a means for efficiently processing strings or sequences.

Use case: A basic tokenizer

Probability - a measurement of how likely an event is to occur.

Use case: How likely is “force” to be a noun?



# Regular Expressions

Patterns to match in a string.

Example:

pattern	example strings	matches
ing	'kicking', 'ingles', 'class'	'kick <u>ing</u> ', ' <u>in</u> gles', 'class'X

# Regular Expressions

Patterns to match in a string.

character class: [] --matches any single character inside brackets

pattern	example strings	matches
ing	'kicking', 'ingles', 'class'	'kick <u>ing</u> ', ' <u>in</u> gles', 'class'X
[sS]bu	'sbu', 'I like Sbu a lot', 'SBU'	

# Regular Expressions

Patterns to match in a string.

character class: [] --matches any single character inside brackets

pattern	example strings	matches
ing	'kicking', 'ingles', 'class'	'kick <u>ing</u> ', ' <u>ing</u> les', 'class'X
[sS]bu	'sbu', 'I like Sbu a lot', 'SBU'	' <u>sbu</u> ', 'I like <u>Sbu</u> a lot', 'SBU'X

# Regular Expressions

Patterns to match in a string.

character class: `[]` --matches any single character inside brackets

character ranges: `[ - ]` -- matches a range of characters according to ascii order

pattern	example strings	matches
<code>ing</code>	'kicking', 'ingles', 'class'	'kick <u>ing</u> ', ' <u>ing</u> les', 'class' <b>X</b>
<code>[sS]bu</code>	'sbu', 'I like Sbu a lot', 'SBU'	' <u>sbu</u> ', 'I like <u>Sbu</u> a lot', 'SBU' <b>X</b>
<code>[A-Z][a-z]</code>	'sbu', 'Sbu' #capital followed by lowercase	
<code>[0-9][MmKk]</code>	'5m', '50m', '2k', '2b'	

# Regular Expressions

Patterns to match in a string.

character class: `[]` --matches any single character inside brackets

character ranges: `[ - ]` -- matches a range of characters according to ascii order

pattern	example strings	matches
<code>ing</code>	'kicking', 'ingles', 'class'	'kick <u>ing</u> ', ' <u>ing</u> les', 'class'X
<code>[sS]bu</code>	'sbu', 'I like Sbu a lot', 'SBU'	' <u>sbu</u> ', 'I like <u>Sbu</u> a lot', 'SBU'X
<code>[A-Z][a-z]</code>	'sbu', 'Sbu' #capital followed by lowercase	'sbu'X, ' <u>Sbu</u> '
<code>[0-9][MmKk]</code>	'5m', '50m', '2k', '2b'	' <u>5m</u> ', '50m'X, ' <u>2k</u> ', '2b'X

# Regular Expressions

Patterns to match in a string.

character class: `[]` --matches any single character inside brackets

character ranges: `[ - ]` -- matches a range of characters according to ascii order

not characters: `[^ ]` -- matches any character except this

pattern	example strings	matches
<code>ing</code>	'kicking', 'ingles', 'class'	'kick <u>ing</u> ', ' <u>ing</u> les', 'class' <b>X</b>
<code>[sS]bu</code>	'sbu', 'I like Sbu a lot', 'SBU'	' <u>sbu</u> ', 'I like <u>Sbu</u> a lot', 'SBU' <b>X</b>
<code>[A-Z][a-z]</code>	'sbu', 'Sbu' #capital followed by lowercase	'sbu' <b>X</b> , ' <u>Sbu</u> '
<code>[0-9][MmKk]</code>	'5m', '50m', '2k', '2b'	' <u>5m</u> ', '50m' <b>X</b> , ' <u>2k</u> ', '2b' <b>X</b>
<code>ing[^s]</code>	'kicking ', 'holdings ', 'ingles '	

# Regular Expressions

Patterns to match in a string.

character class: `[]` --matches any single character inside brackets

character ranges: `[ - ]` -- matches a range of characters according to ascii order

not characters: `[^ ]` -- matches any character except this

pattern	example strings	matches
<code>ing</code>	'kicking', 'ingles', 'class'	'kick <u>ing</u> ', ' <u>ing</u> les', 'class'X
<code>[sS]bu</code>	'sbu', 'I like Sbu a lot', 'SBU'	' <u>sbu</u> ', 'I like <u>Sbu</u> a lot', 'SBU'X
<code>[A-Z][a-z]</code>	'sbu', 'Sbu' #capital followed by lowercase	'sbu'X, ' <u>Sbu</u> '
<code>[0-9][MmKk]</code>	'5m', '50m', '2k', '2b'	' <u>5m</u> ', '50m'X, ' <u>2k</u> ', '2b'X
<code>ing[^s]</code>	'kicking ', 'holdings ', 'ingles ', 'kicking'	'kick <u>ing</u> ', 'holdings 'X, ' <u>ing</u> les', 'kicking'X

# Regular Expressions

Pattern

In python we denote regular expressions with:

`r'PATTERN'`

cha

character rang

according to ascii order

not characters

matches any character except this

pattern	example strings	matches
<code>r'ing'</code>	'kicking', 'ingles', 'class'	'kick <u>ing</u> ', ' <u>ing</u> les', 'class'X
<code>r'[sS]bu'</code>	'sbu', 'I like Sbu a lot', 'SBU'	' <u>sbu</u> ', 'I like <u>Sbu</u> a lot', 'SBU'X
<code>r'[A-Z][a-z]'</code>	'sbu', 'Sbu' #capital followed by lowercase	'sbu'X, ' <u>Sbu</u> '
<code>r'[0-9][MmKk]'</code>	'5m', '50m', '2k', '2b'	' <u>5m</u> ', ' <u>50m</u> ', ' <u>2k</u> ', '2b'X
<code>r'ing[^s]'</code>	'kicking ', 'holdings ', 'ingles '	'kick <u>ing</u> ', 'holdings 'X, ' <u>ing</u> les '



# Regular Expressions

Matching recurring patterns:

\* : match 0 or more

+ : match 1 or more

pattern	example strings	matches
<code>r'ing!*</code>	<code>'swing', 'swing!' 'swing!!!' '!!!'</code>	
<code>r'[sS][oO]+'</code>	<code>'so', 'sooo', 'SOOoo', 'so!', 'soso'</code>	

# Regular Expressions

Matching recurring patterns:

\* : match 0 or more

+ : match 1 or more

pattern	example strings	matches
r'ing!*	'swing', 'swing!' 'swing!!!' '!!!'	' <u>swing</u> ', ' <u>swing!</u> ' ' <u>swing!!!</u> ' '!!!'X
r'[sS][oO]+'	'so', 'sooo', 'SOOoo', 'so!', 'soso'	' <u>so</u> ', ' <u>sooo</u> ', ' <u>SOOoo</u> ', ' <u>so!</u> ', ' <u>so</u> ' <u>so</u> ' #would match twice

# Regular Expressions

Matching recurring patterns:

\* : match 0 or more

+ : match 1 or more

? : 0 or 1

pattern	example strings	matches
r'ing!*	'swing', 'swing!' 'swing!!!' '!!!'	' <u>swing</u> ', ' <u>swing!</u> ' ' <u>swing!!!</u> ' '!!!'X
r'[sS][oO]+'	'so', 'sooo', 'SOOoo', 'so!', 'soso'	' <u>so</u> ', ' <u>sooo</u> ', ' <u>SOOoo</u> ', ' <u>so!</u> ', ' <u>so</u> ' <u>so</u> ' #would match twice
r'oranges?'	'orange', 'oranges', 'orangess'	

# Regular Expressions

Matching recurring patterns:

\* : match 0 or more

+ : match 1 or more

? : 0 or 1

pattern	example strings	matches
r'ing!*	'swing', 'swing!' 'swing!!!' '!!!'	' <u>swing</u> ', ' <u>swing!</u> ' ' <u>swing!!!</u> ' '!!!'X
r'[sS][oO]+'	'so', 'sooo', 'SOOoo', 'so!', 'soso'	' <u>so</u> ', ' <u>sooo</u> ', ' <u>SOOoo</u> ', ' <u>so!</u> ', ' <u>so</u> ' <u>so</u> ' #would match twice
r'oranges?'	'orange', 'oranges', 'orangess'	' <u>orange</u> ', ' <u>oranges</u> ', ' <u>orangess</u> ' #matches all it can

# Regular Expressions

Patterns applied to groups of characters

AA|BB : matches group AA or group BB

pattern	example strings	matches
r'hers his theirs"	'this is hers', 'this is his!'	'this is <u>hers</u> ', 'this is <u>his</u> !'

# Regular Expressions

Patterns applied to groups of characters

AA|BB : matches group AA or group BB

(AA) : apply any following operations to group

pattern	example strings	matches
r'hers his'	'this is hers', 'this is his!'	'this is <u>hers</u> ', 'this is <u>his!</u> '
r'([A-Z][a-z]+ )+'	'This matches Cap Words followed By a Space.'	

# Regular Expressions

Patterns applied to groups of characters

AA|BB : matches group AA or group BB

(AA) : apply any following operations to group

pattern	example strings	matches
r'hers his'	'this is hers', 'this is his!'	'this is <u>hers</u> ', 'this is <u>his!</u> '
r'([A-Z][a-z]+ )+'	'This matches Cap Words followed By a Space.'	' <u>This</u> matches <u>Cap Words</u> followed <u>By</u> a Space.'

# Regular Expressions

. : any single character

pattern	example strings	matches
.	'kicking'	' <u>k</u> ' ' <u>i</u> ' ' <u>c</u> ' ' <u>k</u> ' ...



# Regular Expressions

. : any single character

\$ : end of string

pattern	example strings	matches
.	'kicking'	' <u>k</u> ' ' <u>i</u> ' ' <u>c</u> ' ' <u>k</u> '
.\$	'great', 'great!', '50'	

# Regular Expressions

. : any single character

\$ : end of string

pattern	example strings	matches
.	'kicking'	' <u>k</u> ' ' <u>i</u> ' ' <u>c</u> ' ' <u>k</u> '
.\$	'great', 'great!', '50'	'great <u>t</u> ', 'great! <u>!</u> ', '5 <u>0</u> '

# Regular Expressions

. : any single character

\$ : end of string

^: beginning of string

pattern	example strings	matches
.	'kicking'	' <u>k</u> ' ' <u>i</u> ' ' <u>c</u> ' ' <u>k</u> '
.\$	'great', 'great!', '50'	'great <u>t</u> ', 'great <u>!</u> ', '5 <u>0</u> '
^a	'Happy', 'slate', 'a', 'kick a door'	

# Regular Expressions

. : any single character

\$ : end of string

^: beginning of string

pattern	example strings	matches
.	'kicking'	' <u>k</u> ' ' <u>i</u> ' ' <u>c</u> ' ' <u>k</u> '
.\$	'great', 'great!', '50'	'great <u>t</u> ', 'great! <u>!</u> ', '5 <u>0</u> '
^a	'Happy', 'slate', 'a', 'kick a door'	' <u>H</u> appy', 'slate', 'a'X, 'kick a door'
.a	'Happy', 'slate', 'a', 'kick a door'	' <u>H</u> appy', 'sl <u>a</u> te', 'a'X, 'kick <u>a</u> door'

# Regular Expressions

`\s` : matches any whitespace (space, tab, newline)

`\b` : matches a word boundary

Tokenizing -- breaking a sentence into simple lexical units (basically words).

Here are a couple simple regular expressions for tokenizing:

pattern	example strings	matches
<code>r'(\s ^[A-z]+...</code>	'Kick a door.'	

# Regular Expressions

`\s` : matches any whitespace (space, tab, newline)

`\b` : matches a word boundary

Tokenizing -- breaking a sentence into simple lexical units (basically words).

Here are a couple simple regular expressions for tokenizing:

pattern	example strings	matches
<code>r'(\s ^)[A-z]+([!?\.\.])?\$'</code>	'Kick a door.'	

# Regular Expressions

`\s` : matches any whitespace (space, tab, newline)

`\b` : matches a word boundary

Tokenizing -- breaking a sentence into simple lexical units (basically words).

Here are a couple simple regular expressions for tokenizing:

pattern	example strings	matches
<code>r'(\s ^)[A-z]+([!?\.\.])?\$'</code>	'Kick a door.'	'Kick' ' a' ' door.'

# Regular Expressions

`\s` : matches any whitespace (space, tab, newline)

`\b` : matches a word boundary

Tokenizing -- breaking a sentence into simple lexical units (basically words).

Here are a couple simple regular expressions for tokenizing:

pattern	example strings	matches
<code>r'(\s ^)[A-z]+([!?\.\.])\$)?'</code>	'Kick a door.'	' <u>Kick</u> ' ' <u>a</u> ' <u>door.</u> '
<code>r'\b[A-z]+\b'</code>	'Kick a door.'	' <u>Kick</u> <u>a</u> <u>door.</u> ' #3 matches, no whitespace



# Regular Expressions

```
import re

words = re.findall(r'\b[A-z]+\b', sentence)

for word in words:
    print(word)
```

pattern	example strings	matches
<code>r'(\s ^)[A-z]+([!?\.\.])\$)?'</code>	'Kick a door.'	' <u>Kick</u> ' ' <u>a</u> ' ' <u>door.</u> '
<code>r'\b[A-z]+\b'</code>	'Kick a door.'	' <u>Kick a door.</u> ' #3 matches, no whitespace

# What is Probability?

## Examples

1. outcome of flipping a coin
2. side of a die
3. mentioning a word
4. mentioning a word “a lot”

# What is Probability?

The chance that something will happen.

Given infinite observations of an event, the proportion of observations where a given outcome happens.

Strength of belief that something is true.

“Mathematical language for quantifying uncertainty” - Wasserman

# Probability

$\Omega$  : Sample Space, set of all outcomes of a random experiment

$A$  : Event ( $A \subseteq \Omega$ ), collection of possible outcomes of an experiment

$P(A)$ : Probability of event  $A$ ,  $P$  is a function: events  $\rightarrow \mathbb{R}$

# Probability

$\Omega$  : Sample Space, set of all outcomes of a random experiment

$A$  : Event ( $A \subseteq \Omega$ ), collection of possible outcomes of an experiment

$P(A)$ : Probability of event  $A$ ,  $P$  is a function: events  $\rightarrow \mathbb{R}$

1.  $P(\Omega) = 1$
2.  $P(A) \geq 0$  , for all  $A$

If  $A_1, A_2, \dots$  are disjoint events then:

$$P\left(\bigcup_i A_i\right) = \sum_i P(A_i)$$

# Probability

$\Omega$  : Sample Space, set of all outcomes of a random experiment

$A$  : Event ( $A \subseteq \Omega$ ), collection of possible outcomes of an experiment

$P(A)$ : Probability of event  $A$ ,  $P$  is a function: events  $\rightarrow \mathbb{R}$

$P$  is a *probability measure*, if and only if

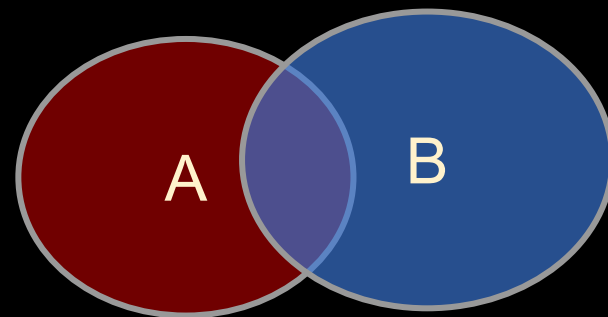
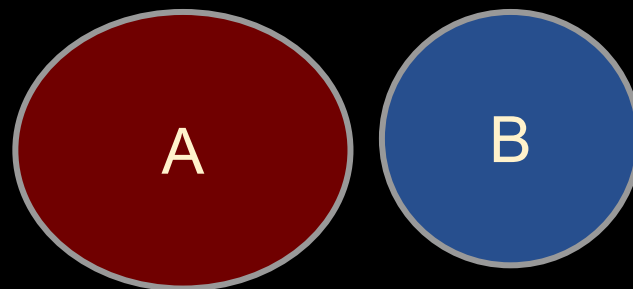
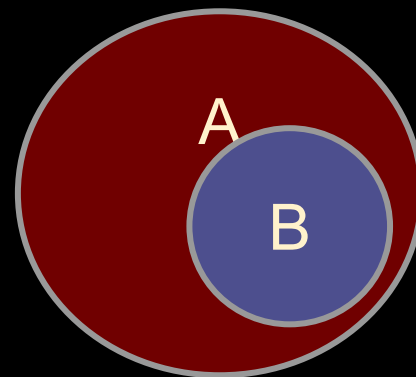
1.  $P(\Omega) = 1$
2.  $P(A) \geq 0$  , for all  $A$

If  $A_1, A_2, \dots$  are disjoint events then:

$$P\left(\bigcup_i A_i\right) = \sum_i P(A_i)$$

# Probability

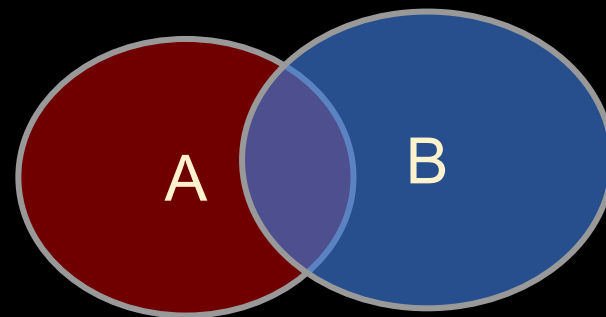
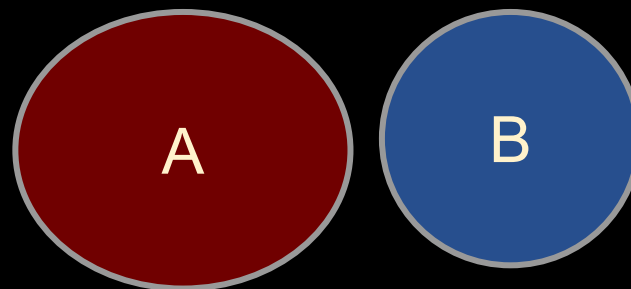
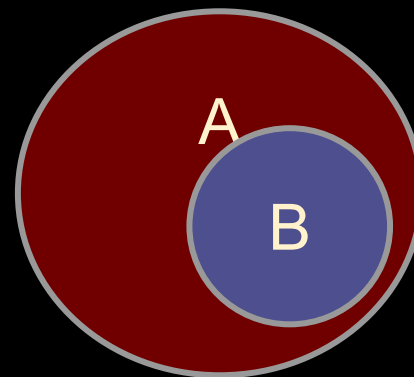
Some Properties:



# Probability

## Some Properties:

1. If  $B \subseteq A$  then  $P(A) \geq P(B)$

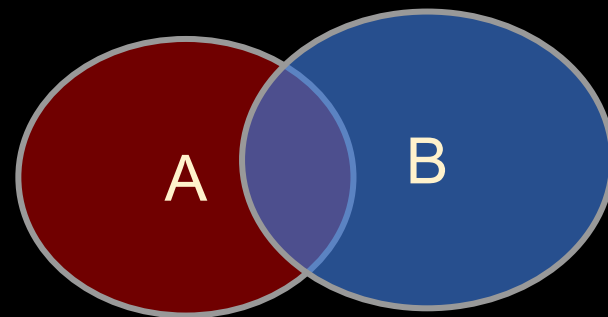
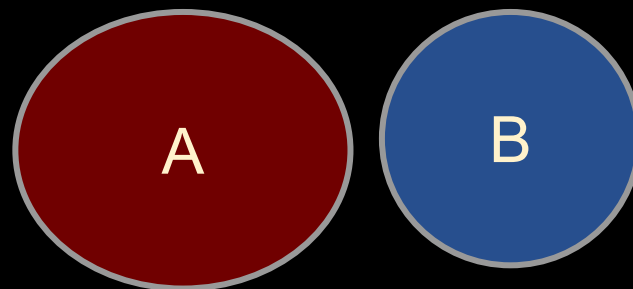
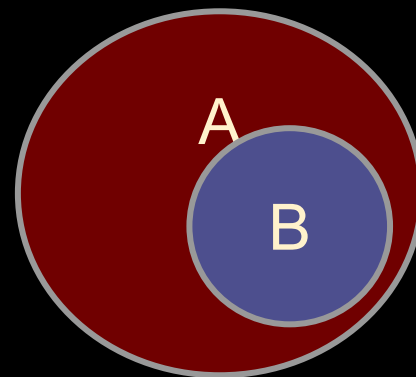




# Probability

## Some Properties:

1. If  $B \subseteq A$  then  $P(A) \geq P(B)$
2.  $P(A \cup B) \leq P(A) + P(B)$



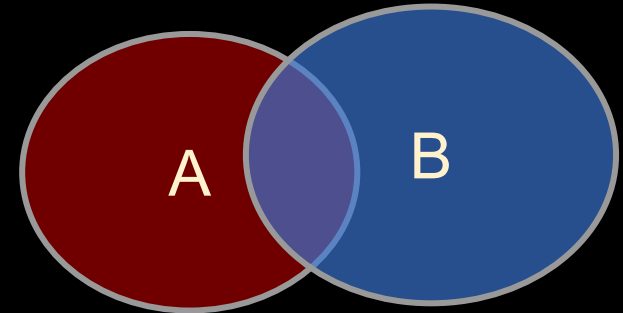
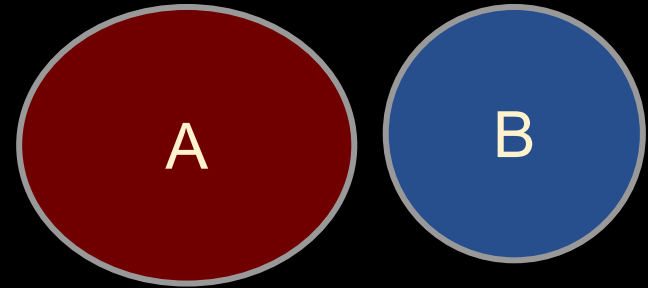
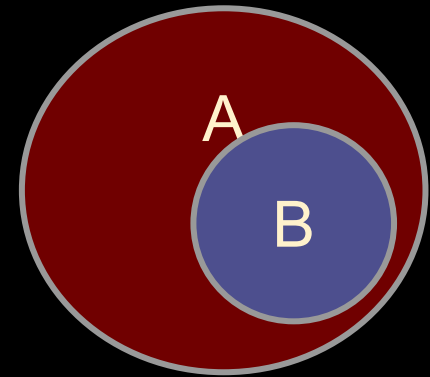
# Probability

## Some Properties:

1. If  $B \subseteq A$  then  $P(A) \geq P(B)$
2.  $P(A \cup B) \leq P(A) + P(B)$
3.  $P(A \cap B) \leq \min(P(A), P(B))$
4.  $P(\neg A) = P(\Omega / A) = 1 - P(A)$

$/$  is set difference

$P(A \cap B)$  will be notated as  $P(A, B)$



# Probability

## Independence

Two Events:  $A$  and  $B$

Does knowing something about  $A$  tell us whether  $B$  happens (and vice versa)?

# Probability

## Independence

Two Events:  $A$  and  $B$

Does knowing something about  $A$  tell us whether  $B$  happens (and vice versa)?

1.  $A$ : first flip of a fair coin;  $B$ : second flip of the same fair coin
2.  $A$ : mention or not of the word “happy”  
 $B$ : mention or not of the word “birthday”

# Probability

## Independence

Two Events:  $A$  and  $B$

Does knowing something about  $A$  tell us whether  $B$  happens (and vice versa)?

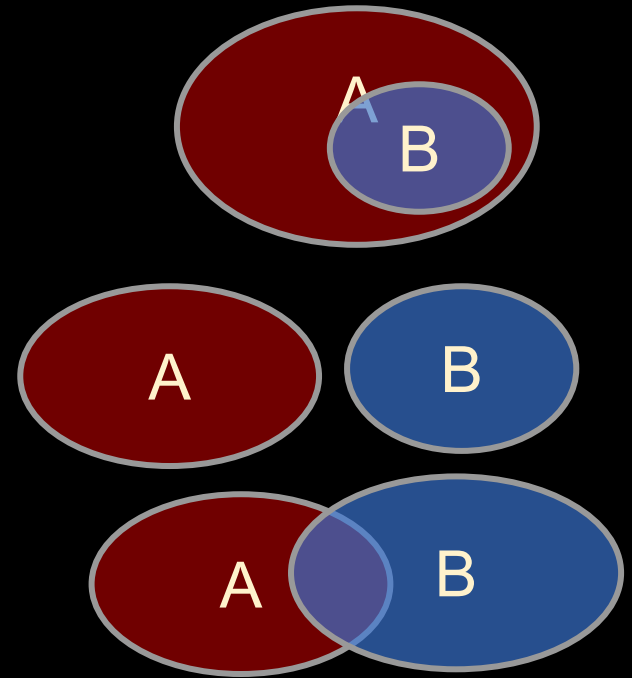
1.  $A$ : first flip of a fair coin;  $B$ : second flip of the same fair coin
2.  $A$ : mention or not of the word “happy”  
 $B$ : mention or not of the word “birthday”

Two events,  $A$  and  $B$ , are *independent* iff:  $P(A, B) = P(A)P(B)$

# Probability

## Conditional Probability

$$P(A|B) = \frac{P(A, B)}{P(B)}$$



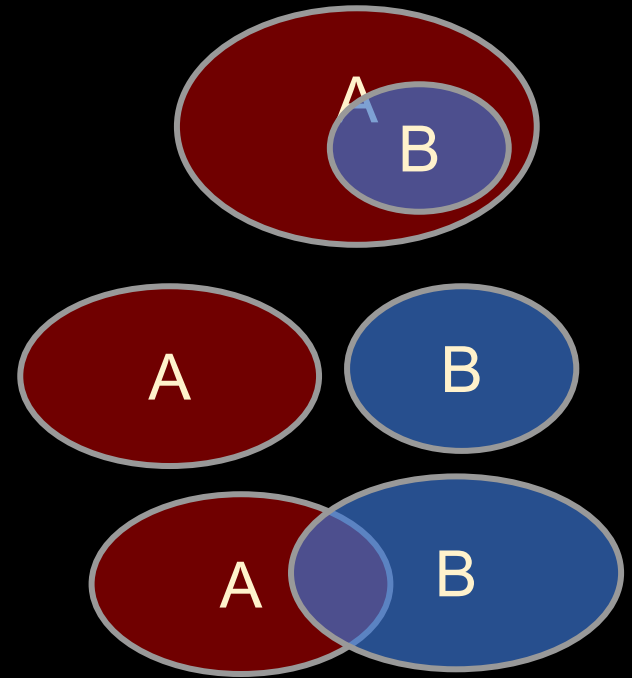
# Probability

## Conditional Probability

$$P(A|B) = \frac{P(A, B)}{P(B)}$$

“|” is often referred to as “given”:

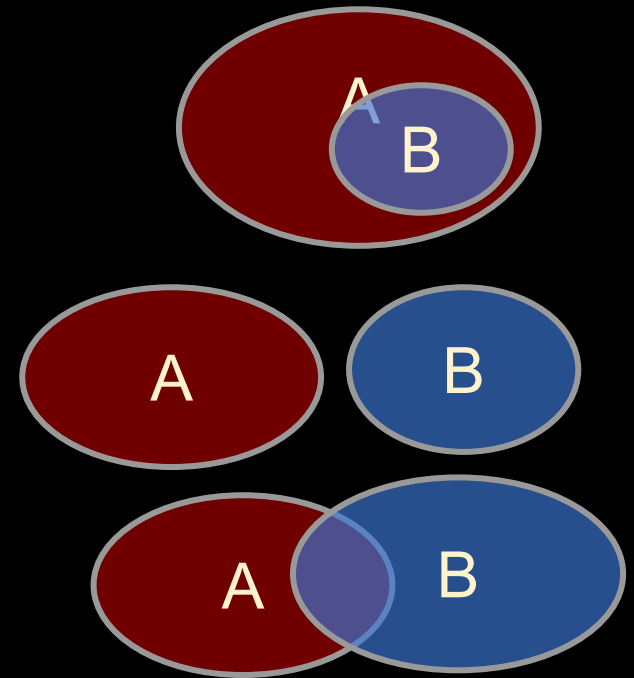
*“The probability of A **given** B is ...”*



# Probability

## Conditional Probability

$$P(A|B) = \frac{P(A, B)}{P(B)}$$



Two events, A and B, are *independent* iff:  $P(A, B) = P(A)P(B)$

$P(A, B) = P(A)P(B)$  iff  $P(B|A) = P(B)$

Interpretation of Independence:

Observing *A* has no effect on probability of *B*.

(Disjoint events, typically, are not independent!)



# Probability

## Conditional Probability

$$P(A|B) = \frac{P(A, B)}{P(B)}$$

### Independence example:

F1=H: first flip of a fair coin is heads

F2=H: second flip of the same coin is heads

$$P(F1=H) = 0.5 \quad P(F2=H) = 0.5$$

$$P(F2=H, F1=H) = 0.25$$

$$P(F2=H|F1=H) = 0.5 = P(H2)$$

Two events, A and B, are *independent* iff:  $P(A, B) = P(A)P(B)$

$$P(A, B) = P(A)P(B) \text{ iff } P(B|A) = P(B)$$

Interpretation of Independence:

Observing A has no effect on probability of B. (and vice-versa)

# Probability

## Conditional Probability

$$P(A|B) = \frac{P(A, B)}{P(B)}$$

### Dependence example:

W1=happy: first word is “happy”

W2=birthday: second word is “birthday”

*from observing language data, we find:*

$$P(W1=happy) = 0.1, P(W2=birthday) = 0.05$$

$$P(W1=happy, W2=birthday) = 0.025$$

*thus*  $P(A, B) \neq P(A)P(B)$

*also*  $P(B|A) \neq P(B)$ :

$$P(W2=birthday|W1=happy) = .025 / .1 = .25 \neq 0.05 = P(W2=birthday)$$

Two events, A and B, are *independent* iff:  $P(A, B) = P(A)P(B)$

$P(A, B) = P(A)P(B)$  iff  $P(B|A) = P(B)$

Interpretation of Independence:

Observing A has no effect on probability of B. (and vice-versa)

# Why Probability?

A formality to make sense of the world.

1. To quantify uncertainty in language data.  
*Should we believe something or not? Is it a meaningful difference?*
2. To be able to generalize from one situation to another.  
*Can we rely on some information? What is the chance Y happens?*
3. To create structured data.  
*Where does X belong? What words are similar to X?*  
*(necessary no matter what approaches take place)*